AN APPLICATION OF THE SECOND MICROLOCALIZATION AT THE BOUNDARY TO THE EXTENSION OF SOLUTIONS OF DIFFERENTIAL SYSTEMS(Functional-Analytic Study of Generalized Functions)

数理解析研究所講究録
704 巻 1989 年 28-36

AN APPLICATION OF THE SECOND MICROLOCALIZATION AT THE BOUNDARY TO THE EXTENSION OF SOLUTIONS OF DIFFERENTIAL SYSTEMS

Giuseppe Zampieri (Padova Univ.)

ABSTRACT We use the theory of microlocalization of sheaves of [K-S-2], and especially its formulation in [S] for boundary value problems to treat the extension of regular solutions of systems of P.D.E. across an 1-codimensional singular set. Let M be a real analytic manifold, X a complexification of M, N an analytic hypersurface of M, Ω an open component of $M \setminus N$. For a suitable involutive manifold $V \subset T^*_M X$, invariant under the Hamiltonian flow of $N \underset{M}{\times} T^*_M X$, we introduce a new complex $\mathcal{B}^a_{\Omega|X}$ of hyperfunctions in Ω with real analytic parameters and study its applications to non-characteristic boundary value problems. In particular we show that the trace morphism preserves the analytic parameters. The analysis of $\mathcal{B}^a_{\Omega|X}$ could be performed from the viewpoint of the 2nd microlocalization at the boundary along V developped in [U-Z]; but we do not need to refer to such a general theory for the purpose of the present paper. We then consider a differential system $\mathcal{M}$ at x, $x \in N$, and a closed set S, $S \subset N$, $x \in \partial S$. We denote by $\tilde{V}$ the union of the leaves of $V^{\mathbb{C}}$ issued from V, we let ρ be the projection $Y \underset{X}{\times} T^*X \to T^*Y$, and make the following hypotheses: the conormals to N at x are non-microcharacteristic for $\mathcal{M}$ along $\tilde{V}$ in $\dot{\pi}^{-1}(x)$; $\mathrm{char}\,\mathcal{M} \cap \rho^{-1}\rho(\{x\} \underset{M}{\times} V) \subset V$; $i N^*_x(S) \subset \rho(\{x\} \underset{M}{\times} V)$. We then prove that $H^0(\mathcal{B}^a_{M|X})$-solutions of $\mathcal{M}$ on $M \setminus S$ extend to M at x. Under some additional assumptions on "propagation in the interior" we also obtain the extension of $\mathcal{A}_M$-solutions. We refer to [Kan], [Ô], and [U-Z] for other results on continuation of (regular) solutions.

§ 1. THE COMPLEXES $\mathcal{B}^2_{\Omega|X}$ AND $\mathcal{B}^a_{\Omega|X}$

Let $M = M' \times L$ be real analytic manifolds with complexifications $X = X' \times Z$ and dimensions $n = n_1 + n_2$. For a locally closed set $A = A'\times L$ of M, put $\tilde{A} = A'\times Z$ and define (cf [K-S-2], [S])

$$(1.1) \qquad \mathcal{C}^h_{A|X} = \mu_{\tilde{A}}(\mathcal{O}_X) \otimes \omega_{M'/X'}[n_1] \quad ,$$

$$(1.2) \qquad \mathcal{B}^2_{A|X} = \mathrm{R}\,\Gamma_{T^*X'\times L}(\mathcal{C}^h_{A|X}) \otimes \omega_{L/Z}[n_2] \quad .$$

We often consider the case $A = M$ or $A = N$ for an analytic submanifold $N = N'\times L$ of M of codimension 1, or else $A = \Omega$ where $\Omega = \Omega^{\pm}$ are the components of $M \setminus N$. The following triangle will play an essential role:

$$(1.3) \qquad \mathcal{B}^2_{N|X} \to \mathcal{B}^2_{M|X} \to \mathcal{B}^2_{\Omega^+|X} \oplus \mathcal{B}^2_{\Omega^-|X} \xrightarrow{+1} \quad .$$

REMARK 1.1. By the results of [U-Z] we could give a canonical definition of the complexes $\mathcal{C}^h_{*|X}$ and $\mathcal{B}^2_{*|X}$, $* = M, N, \Omega$, associated to a smooth conic regular involutive manifold $V \subset \dot{T}^*_M X$ such that

$$(1.4) \qquad V \text{ and } N \underset{M}{\times} T^*_M X \text{ intersect transversally, and } N \underset{M}{\times} V \text{ is regular involutive} \quad .$$

We recall from [K-L] that for $* = M, N$, $\mathcal{B}^2_{*|X}$ is concentrated in degree 0 and the natural morphism $\mathcal{C}_{*|X}|_{T^*X'\times L} \to \mathcal{B}^2_{*|X}$ is injective, $\mathcal{C}_{*|X}$ being the sheaf of usual microfunctions. (As for the case $* = \Omega$ it is proven in [U-Z] that $(\mathcal{B}^2_{\Omega|X})_{T^*_{M'}X'\times L}$ is concentrated in degree 0 but that the corresponding result on injectivity does not hold any more. However this is not needed here.)

We set now:

$$(1.5) \qquad \mathcal{B}^a_{*|X} = \mathrm{R}\,\Gamma_M(\mathcal{O}_X|_{\bar{*}\times Z}) \otimes \omega_{L/Z}[n_2], \qquad * = M, N, \Omega \,.$$

For $* = M, \Omega$ we have a distinguished triangle

$$(1.6) \qquad \mathcal{B}^a_{*|X} \to \mathrm{R}\,\Gamma_*(\mathcal{B}_M) \to \mathrm{R}\,\dot{\pi}_*(\mathcal{B}^2_{*|X}) \xrightarrow{+1} \quad ,$$

($\mathcal{B}_M$ being the sheaf of hyperfunctions); for $* = N$ we have to shift by -1 the first term of (1.6). Using (1.6), the results of [K-L], (and also the trick of the dummy variable for $* = N$), one easily sees that $\mathcal{B}^a_{*|X}$, $* = M$ or N, are concentrated in degrees 0 and 1 with $H^1(\mathcal{B}^a_{*|X}) \neq 0$. The same should be proven for $* = \Omega$; but this is complicated and needless here.

The detailed study of the complexes (1.5) is left to [U-Z]; we only treat here their applications to boundary value problems. Thus let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module on an open set of M. We assume all through this section that Y, the complexification of N, is non-characteristic for $\mathcal{M}$.

PROPOSITION 1.2. The natural morphisms

(1.7) $$H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{C}_{\Omega|X}))|_{T^*X'\times L} \to H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^2_{\Omega|X}))\ ,$$

and

(1.8) $$H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^a_{\Omega|X})) \to \mathcal{H}om(\mathcal{M}\,,\ \Gamma_\Omega(\mathcal{B}_M))\ ,$$

are injective.

PROOF. By the results of [K-L] it is enough to prove (1.7) and (1.8) in $T^*_{N'}X'\times L$ and N respectively. As for (1.7), set $F = T^*_MX \underset{M}{\oplus} N^*(\Omega)^a$ ("a"= antipodal) and consider the commuting diagram

(1.9)
$$\begin{array}{ccccc} & \nearrow & \mathcal{C}_{\Omega|X} & \to & \mathcal{B}^2_{\Omega|X} \\ \mathbf{R}\,\Gamma_F\,\mathcal{C}_{N|X}[1] & & & & \downarrow \\ & \searrow & \mathcal{C}_{N|X}[1] & \to & \mathcal{B}^2_{N|X}[1]\ . \end{array}$$

Then the conclusion follows from:

(1.10) $$\mathcal{H}om(\mathcal{M}\,,\ \mathcal{C}_{N|X}) = 0, \qquad \mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^2_{N|X}/\mathcal{C}_{N|X})\Big|_{T^*_{N'}X'\times L} = 0,$$

which are in turn easy consequences of division formulas for $\mathcal{C}_{N|X}$ and $\mathcal{B}^2_{N|X}$ (cf [K-S-1]).

As for (1.8) we only need to recall (1.6) for $* = \Omega$, and use (1.3), and (1.10). The proof is complete.

Let $\mathcal{M}_Y$ denote the induced system by $\mathcal{M}$ on Y and let $\gamma : \mathcal{H}om(\mathcal{M}, \Gamma_\Omega(\mathcal{B}_M)) \to$ $\to \mathcal{H}om(\mathcal{M}_Y, \mathcal{B}_N)$ be the trace morphism (cf [S]). By collecting all above results we get:

PROPOSITION 1.3. We have

(1.11) $$H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^a_{\Omega|X}))_x = \{u \in \mathcal{H}om(\mathcal{M}\,,\ \Gamma_\Omega(\mathcal{B}_M))_x\ :$$

$$SS(\gamma(u))\cap(T^*_{N'}Y' \times L) \subset T^*_Y Y\ \}, \qquad x \in M.$$

PROOF. Let $\mathcal{F} = \mathbf{R}\,\mathcal{H}om(\mathcal{M}, \mathcal{O}_X)$, put $\tilde{\Omega} = \Omega' \times Z$, and note that the natural diagram

$$(1.12) \qquad \begin{array}{ccc} \pi^{-1}\,\mathbf{R}\Gamma_\Omega(\mathcal{F}) & \to & \mathbf{R}\,\Gamma_{T^*X' \times L}\mu_{\tilde{\Omega}}(\mathcal{F}) \\ & \searrow \quad \mu_\Omega(\mathcal{F})|_{T^*X' \times L} \quad \nearrow & \end{array},$$

is commuting. Thus recalling (1.6) and applying Proposition 1.2, we get

$$(1.13) \qquad H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}, \mathcal{B}^a_{\Omega|X}))_x = \{ u \in \mathcal{H}om(\mathcal{M}, \Gamma_\Omega(\mathcal{B}_M))_x : SS^{\mathcal{M},0}_\Omega(u)\cap(T^*X' \times L) \subset T^*_X X \ \},$$

where $SS^{\mathcal{M},0}_\Omega(u)$ is the support of u identified to a section of $H^0(\mathbf{R}\,\mathcal{H}om(\mathcal{M}, \mathcal{C}_{\Omega|X}))$ (cf [S]). According to [S] this is in turn equivalent to (1.11).

REMARK 1.4. When considering $\mathcal{B}^a_{M|X}$ one can use the injectivity of $\mathcal{C}_{M|X}|_{T^*_{M'}X' \times L} \to \mathcal{B}^2_{M|X}$ and $H^0(\mathcal{B}^a_{M|X}) \to \mathcal{B}_M$ as a substitute of Proposition 1.2. (Note that the latter injectivity follows from (1.6) and the (conical) flabbiness of $\mathcal{B}^2_{M|X}$ (cf [K-L]).) Then using (1.12) one easily gets

$$(1.14) \qquad H^0(\mathcal{B}^a_{M|X})_x = \{ u \in (\mathcal{B}_M)_x : SS(u)\cap(T^*_{M'}X' \times L) \subset T^*_X X\ \}, \qquad x \in M.$$

REMARK 1.5. For a regular involutive manifold V defined on the whole $T^*_M X$ and satisfying (1.4), we can intrinsecally define $\mathcal{B}^a_{*|X}$, $* = M, N, \Omega$, by replacing in (1.5) $\bar{*} \times Z$ by $\pi(\tilde{V}_{\bar{*}})$ (and $\omega_{L/Z}$ by $\omega_{V/\tilde{V}_M}$), where $\tilde{V}_{\bar{*}}$ is the union of the leaves of $V^{\mathbf{C}}$ issued from $\bar{*} \underset{M}{\times} T^*_M X$; (we also write $\tilde{V} = \tilde{V}_M$). One can also intrinsecally define the right hand sides of (1.11), (1.14) just by replacing $T^*_{N'}Y' \times L$ and $T^*_{M'}X' \times L$ by $\rho\varpi^{-1}(V)$ and V respectively (ρ and ϖ being the natural mappings from $Y \underset{X}{\times} T^*X$ to T^*Y and T^*X resp.).
It is then clear that if for some coordinates on M we can write

$$V = T^*_{M'}X' \times L, \qquad N = N' \times L\ ,$$

then (1.11) and (1.14) still hold. More generally owing to the invariance of $\mathcal{B}^2_{*|X}$ under contact transformation preserving V, $N \underset{M}{\times} V$, and $\omega_{N/M}$ (cf [U-Z]), one could prove that (1.6) is fulfilled. But this refined argument is not needed here.

§ 2. EXTENSION OF SOLUTIONS WITH REAL ANALYTIC PARAMETERS

Let M be a real analytic manifold with complexification X, N an analytic hypersurface of M with complexification Y, $\Omega = \Omega^{\pm}$ the two components of $M \setminus N$, ρ and ϖ the canonical mappings from $Y \underset{X}{\times} T^*X$ to T^*Y and T^*X respectively. Let $x \in M$, let $U \subset M$ be a neighborhood of x, and let V be a manifold in $U \underset{M}{\times} T^*_M X$. We assume that, in suitable coordinates on U :

$$(2.1) \qquad M = M' \times L, \qquad X = X' \times Z, \qquad N = N' \times L$$
$$V = T^*_{M'}X' \times L, \qquad \tilde{V} = T^*_{M'}X' \times Z \;.$$

Recall the complexes $\mathcal{B}^a_{M|X}$, $\mathcal{B}^a_{\Omega|X}$ (intrinsecally associated to V) and remember (1.11), (1.14). For any $p \in \dot{\pi}^{-1}(x)$ recall the identification $T^*_x M \hookrightarrow T_p T^*X$ obtained through the embedding $T^*X \underset{X}{\times} T^*X \hookrightarrow T^*T^*X$ and the Hamiltonian isomorphism, and observe that $(T^*_N M)_x/\mathbf{R}^+$ is just a pair of vectors $\pm\,\theta$.

THEOREM 2.1. <u>Let</u> N <u>and</u> V <u>be defined, in suitable coordinates by</u> (2.1), <u>and let</u> $\mathcal{M}$ <u>be a coherent</u> $\mathcal{D}_X$<u>-module at</u> x <u>which verifies</u>

$$(2.2) \qquad \pm\theta \notin C_p(\operatorname{char}\mathcal{M}, \tilde{V}) \quad \underline{\text{for}}\ \pm\theta \in (T^*_N M)_x/\mathbf{R}^+\ \underline{\text{and for any}}\ p \in \dot{\pi}^{-1}(x) \cap V,$$

$$(2.3) \qquad \varpi^{-1}(\operatorname{char}\mathcal{M}) \cap \rho^{-1}\rho(\{x\} \underset{M}{\times} V) \subset T^*_M X \;.$$

<u>Let</u> S <u>be a closed subset of</u> N <u>with</u> $x \in \partial S$ <u>and</u>

$$(2.4) \qquad i\,N^*_x(S) \subset \rho\varpi^{-1}(V)_x \;,$$

<u>(in the identification</u> $i\,T^*N \simeq T^*_N Y$). <u>We then have, in a neighborhood of</u> x,

$$(2.5) \qquad \mathcal{H}om(\mathcal{M}, \Gamma_{M \setminus S}(H^0(\mathcal{B}^a_{M|X}))) \xleftarrow{\sim} \mathcal{H}om(\mathcal{M}, H^0(\mathcal{B}^a_{M|X})) \;.$$

PROOF. Let $\Omega = \Omega^{\pm}$ with $\Omega^+ \cup \Omega^- = M \setminus N$; by reasoning as in § 1 and observing that

$$\mathbf{R}\,\pi_*\,\mathbf{R}\,\Gamma_{(T^*_X X \cup \dot{\pi}^{-1}(N))}(\mathcal{B}^2_{\Omega|X}) \;=\; \mathbf{R}\,\Gamma_{\Omega}(\mathcal{B}^a_{M|X}) \;,$$

we get a distinguished triangle

$$(2.6) \qquad \mathcal{B}^a_{\Omega|X} \to \mathbf{R}\,\Gamma_{\Omega}(\mathcal{B}^a_{M|X}) \to \mathbf{R}\,\dot{\pi}_*\,\mathbf{R}\,\Gamma_{\dot{\pi}^{-1}(N)}(\mathcal{B}^2_{\Omega|X}) \xrightarrow{+1}$$

Let $\mathcal{F} = \mathbf{R}\,\mathcal{H}om(\mathcal{M}, \mathcal{C}^h_{\Omega|X})|_{M \underset{X}{\times} T^*X}$. We note that (2.2) implies $(p;\pm\theta) \notin SS(\mathcal{F})$ and thus also $\mathbf{R}\,\Gamma_{\dot{\pi}^{-1}(N)}(\mathcal{F}) = \mathbf{R}\,\Gamma_{\dot{\pi}^{-1}(M \setminus \Omega)}(\mathcal{F}) = 0$. By applying

$R\,\Gamma_{(N \widetilde{\times_M} V \cap T^*_N X)}(\cdot)$ $[n_2]$ to the last equality ($N \widetilde{\times_M} V$ being defined similarly to $\hat{V}$ and n_2 being the codimension of V), we then get, for a neighborhood U of x on N,

$$(2.7) \qquad R\,\Gamma_{\dot{\pi}^{-1}(N)}\, R\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^2_{\Omega|X})\Big|_{U \times_M V} \;=\; 0.$$

Note now that (2.2), (2.3) imply:

$$\varpi^{-1}(\ \mathrm{char}\ \mathcal{M}\ ) \cap \rho^{-1}\ \rho(U \times_M V) \subset U \times_M V \quad ,$$

which gives, combined with (2.7):

$$(2.8) \qquad R\,\dot{\pi}_*\ R\,\Gamma_{\dot{\pi}^{-1}(N)}\ R\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^2_{\Omega|X})\Big|_U \;=\; 0.$$

By (2.6) this implies:

$$(2.9) \qquad R\,\mathcal{H}om(\mathcal{M}\,,\ \mathcal{B}^a_{\Omega|X})\Big|_U \;\simeq\; R\,\mathcal{H}om(\mathcal{M}\,,\ R\,\Gamma_{\Omega}(\mathcal{B}^a_{M|X})\ )\Big|_U \ .$$

For $u \in \mathcal{H}om(\mathcal{M}, \Gamma_{M\setminus S}(H^0(\mathcal{B}^a_{M|X})))$ let now $u^{\pm} = u|_{\Omega^{\pm}}$. Owing to (2.9) and (1.11) we get

$$SS(\gamma(u^{\pm})) \cap \rho(U \times_M V) \ \subset\ T^*_Y Y\ .$$

We also clearly have

$$\mathrm{supp}(\gamma(u^+) - \gamma(u^-)) \ \subset\ S.$$

Therefore the conclusion is an immediate consequence of the following two lemmas.

LEMMA 2.2 (cf [$\hat{0}$]). <u>Let</u> F <u>be a closed set of</u> M <u>and let</u> $u \in (\mathcal{B}_M)_x$, $x \in \partial F$. <u>Then</u>

$$\begin{cases} SS(u) \cap N^*_x(F) \subset \{0\} \\ \mathrm{supp}(u) \subset F \end{cases} \quad \Leftrightarrow \quad u_x = 0$$

PROOF. Easy application of Kashiwara-Holmgren's theorem and of sweeping out procedure by Bony-Schapira.

LEMMA 2.3. <u>Let</u> $u \in \mathcal{H}om(\mathcal{M}, \Gamma_{(M\setminus N)}(\mathcal{B}_M))$; <u>then</u>

$$u \in \mathcal{H}om(\ \mathcal{M}\,,\ H^0(\mathcal{B}^a_{M|X})\ ) \quad \Leftrightarrow \quad \begin{cases} \gamma(u^{\pm}) \in H^0(\mathcal{B}^a_{N|Y}) \\ \gamma(u^+) - \gamma(u^-) \;=\; 0\ . \end{cases}$$

PROOF. It is enough to recall the triangle

$$\mathcal{C}_{M|X} \to \mathcal{C}_{\Omega^+|X} \oplus \mathcal{C}_{\Omega^-|X} \to \mathcal{C}_{N|X}[1] \xrightarrow{+1},$$

and the estimation

$$SS(u) \subset \bigcup_{\pm} SS^{\mathcal{M},0}_{\Omega^\pm}(u^\pm) \subset \rho^{-1}(\bigcup_{\pm} SS(\gamma(u^\pm))),$$

(cf [S]).

COROLLARY 2.4. In the situation of Theorem 2.1 assume in addition:

(2.10) $\mathcal{H}om(\mathcal{M} , \Gamma_S(\mathcal{C}_{M|X}))_p = 0 \quad \forall p \in \dot{T}^*_M X \setminus V, \ \pi(p) = x.$

Then (for $\mathcal{A}_M = \mathcal{O}_{X|M}$) :

(2.11) $\mathcal{H}om(\mathcal{M} , \Gamma_{(M \setminus S)}(\mathcal{A}_M))_x \simeq \mathcal{H}om(\mathcal{M} , \mathcal{A}_M)_x .$

By the argument in the proof of (2.7) and by the injectivity of $\mathcal{C}_{M|X|V} \to \to \mathcal{B}^2_{M|X}$, a sufficient condition for (2.10) is that (2.2) is fulfilled for some V_p and θ_p such that $p \in V_p$, $S \subset \{x \in M : \langle x, \theta_p \rangle \geq 0\}$.

REMARK 2.5. It is clear from Lemma 2.2 that we can even consider in Theorem 2.1 some singular set S such that $N^*_{x_o}(S) = T^*_{x_o} N$. In fact for $M = M' \times L \simeq \mathbf{R}^{n_1} \times \mathbf{R}^{n_2} \ni x = (x', x'')$, we only need to assume that $N \setminus S$ contains spheres of the L-plane whose diameters are infinite over the distance to ∂S. For example this is the case of any $S \subset \{ \phi \leq 0\}$ for $\phi \in C^0(N)$ with $\phi(x_o) = 0$, $\partial_{x_{n_1}} \phi(x_o) \neq 0$, $\partial_{x''} \phi(x_o) = 0$, $\partial_{x_{n_1}} \phi \in C^0$, $\partial_{x''} \phi \in C^0$.

REMARK 2.6. Theorem 2.1 extends the results of [Kan], [Ô]. These are obtained by choosing $L \simeq \mathbf{R}^{n-2} \subset M \simeq \mathbf{R}^n$ and by replacing $\mathcal{V}$ with $T^*_M X$ in (2.2).

EXAMPLE 2.7. Let $M = M' \times L \ni (x', x'')$, $N = N' \times L$, $M' = \mathbf{R} \times N'$ $x' = (x_1, \tilde{x})$, $S = S' \times L$, $x_o = 0 \in \partial S$. Let (z, ζ), $z = x + iy$, $\zeta = \xi + i\eta$, be coordinates in T^*X, let $V = \{\eta'' = 0\}$ and consider

$$\mathcal{M} \ : \ \zeta_1^2 - (z_1^r + \tilde{z}^s)\tilde{\zeta}^2 + \zeta''^2, \qquad r, \ s \text{ even}, \ r \geq 2.$$

Then (2.2)-(2.4) hold with $\pm\theta = \pm dx_1$ (cf [S-Z]) and thus we get (2.5) and (2.11) (as (2.10) is trivial in the present situation).

EXAMPLE 2.8. In the above situation let $M' \simeq \mathbf{R} \times N' \simeq \mathbf{R} \times \mathbf{R}^3$, let $V = \{\eta_3 = \eta_4 = \eta'' = 0\}$, and consider

$$\mathcal{M} : \left(\zeta_1^3 + \zeta_3^3 + \zeta''^3 , \zeta_2(\zeta_3^2 + \zeta_4^2) \right) .$$

For $S = S' \times L$ with $0 \in \partial S$ we have (2.2)-(2.4) and thus also (2.5). Moreover for any $p \in V$ and for $\pm\theta_p = \pm dx_1$ or $\pm dx_2$ we have (2.2) with $\hat{\gamma}_p = T^*_M X$. Therefore if we let $S = \{x_1 = x_2 = 0\}$, we get (2.10) and (2.11). (This extends Example 1.1 of [Ô].)

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Giuseppe Zampieri
Dep. of Math., Fac. Sci.
Univ. of Tokyo
Tokyo, 113 Japan
and
Dip. Mat. - Università
Padova, 35131 Italy